

Handy Formulas

Primary units in this guide are metric (SI – the International System of units):

- Length - m - (meter)
- Mass - g - (gram)
- Force - mN - (millinewton)
- Torque - mN•m - (millinewton meter)
- Inertia - g•m² - (gram meter²)

In this system, mass is always in kilograms or grams. Force, or weight, is always in newtons or millinewtons.

$$\text{Force (or weight)} = \text{Mass} \times \text{Acceleration}$$

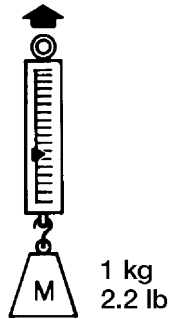
$$F = ma$$

when $a = 9.81 \text{ m/sec}^2$ (acceleration due to gravity), then F would be the weight in newtons.

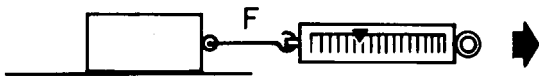
How to measure Mass or Force.

A spring scale reading of 1 kg means that you are measuring a mass of 1 kg.

A spring scale reading of 2.2 lb also is measuring a mass of 1 kg.



If you use that same spring scale to measure a force, the 1 kg reading must be multiplied by 9.8 to give a force of 9.8 newtons.



The reading of 2.2 lb is a force and is equal to 9.8 newtons.

If the same scale is used to measure torque ($T = FR$) at a one meter radius, the reading of

$$1 \text{ kilogram} \times 1 \text{ meter} = 1 \text{ kgm}$$

must be multiplied by 9.8 to give a torque of 9.8 newton meters (N•m).

	Given Unit		Units Used in this Manual (Metric SI)
Length	1 inch =	2.54 cm =	$2.54 \times 10^{-2} \text{m}$
Force	1 oz =		278 mN
	1 lb =	4.45 N =	4,450 mN
	1 g•cm =		9.8 mN
Mass	1 lb =		454g
	1oz =		28.4g
	1kg =		1,000g
	1 slug =	14.6 kg =	14,600g
Inertia	1 g•cm ² =		$10^{-4} \text{g} \cdot \text{m}^2$
	1 oz-in-sec ² =		$7.06 \text{g} \cdot \text{m}^2$
	1 slug ft ² =		$.29 \text{g} \cdot \text{m}^2$
Torque	1 oz-in =	72.01 g•cm =	7.06 mN•m
	1 lb-ft =		$1.356 \times \text{N} \cdot \text{m}$
	1 g•cm =		$9.8 \times 10^{-2} \text{mN} \cdot \text{m}$
		10.2 g•cm =	1 mN•m
		141.6 oz-in =	1 N•m

$$1. \text{ Torque (mN} \cdot \text{m)} = \text{Force (mN)} \times \text{Radius (m)}$$

$$\text{Torque} = FR$$

$$2. \text{ Torque required to accelerate inertial load}$$

$$T \text{ (mN} \cdot \text{m)} = J \alpha$$

$$J = \text{Inertia in } \text{g} \cdot \text{m}^2$$

$$\alpha = \text{Acceleration in radians/sec}^2$$

EXAMPLE:

If a rotor inertia plus load inertia = $J = 2 \times 10^{-3} \text{ g} \cdot \text{m}^2$, and the motor is to be accelerated at 6,000 radians per sec, what torque is required?

$$T = J\alpha = 2 \times 10^{-3} \times 6000$$

$$T = 12 \text{ mN} \cdot \text{m}$$

For stepper motors, α can be converted to radians/sec² from steps/sec².

$$\alpha \text{ (radians/sec)} = \frac{\Delta v \text{ (steps/sec)}}{\Delta t \text{ (accel. time)}} \times \frac{2\pi}{\text{steps/rev}}$$

$$\text{TORQUE} = J \frac{\Delta v}{\Delta t} \times \frac{2\pi}{\text{steps/rev}}$$

EXAMPLE:

For a 48-step per revolution motor accelerating from zero to steps/sec running rate v in Δt seconds.

$$\text{TORQUE} = J \frac{v}{\Delta t} \times \frac{\pi}{24}$$

For information or to place an order in North America: 1 (203) 271-6444 Europe: (44) 1276-691622 Asia: (65) 7474-888

If no *acceleration time* is provided, then a maximum 2-step lag can occur.

$$\Delta t \text{ (sec)} = \frac{2 \text{ (steps)}}{v \text{ (steps/sec)}} \text{ giving the following equation.}$$

$$\text{TORQUE} = J \frac{V^2}{2} \times \frac{2 \pi}{\text{steps/rev}}$$

3. Moment of Inertia

Disc or shaft

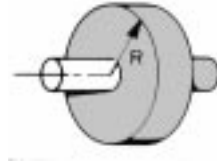
M = Mass in grams

R = Radius in meters

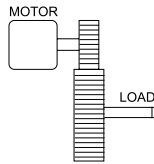
$$J \text{ (g}\cdot\text{m}^2) = \frac{MR^2}{2}$$

Cylinder

$$J \text{ (g}\cdot\text{m}^2) = \frac{M^2}{2} (R_1^2 + R_2^2)$$



4. Reflected loads when using gears or pulleys



$$\text{Torque required of motor} = \frac{\text{Load Torque}}{GR}$$

$$\text{gear or pulley ratio GR} = \frac{\text{motor shaft revolutions}}{\text{load shaft revolutions}}$$

$$\text{Inertia reflected to motor} = \frac{\text{Load inertia}}{(GR)^2}$$

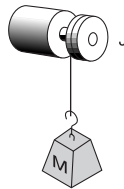
5. Equivalent Inertial Load

For a pulley and weight or a rack and pinion

$$J \text{ eqv. (g}\cdot\text{m}^2) = MR^2$$

M = Mass of load in grams

R = Radius of pulley in meters



6. Total Load

Note: Be sure to include all load components.

$$J_T = \text{Rotor Inertia} + \text{all J Loads}$$

$$T_F = \text{Frictional and Forces}$$

Note: In the pulley example above, the total load would be:

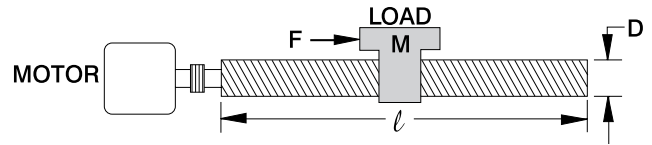
$$J_T = J \text{ rotor} + J \text{ pulley} + J \text{ eqv.}$$

$$T_F = T \text{ frictional} + \text{Load Weight} \times \text{Radius}$$

$$\text{Total T} = J_T \alpha + T_F$$

The load weight = mass x 9.8 millinewtons.

7. Axial Force of Lead Screw



$$F = \frac{2 \pi \times T}{L} \times \text{eff.}$$

F (mN) when T = Torque in mN•m

L = Lead of screw in meters

F (oz) when T = Torque in oz-in

L = Lead of screw in inches

efficiency = from .9 for ballnut to .3 for Acme

Inertia of lead screw load

$$J = J \text{ rotor} + J \text{ steel screw} + J \text{ reflected}$$

$$J \text{ steel screw} = D^4 \times l \times \frac{\pi}{32} \times \text{Density}$$

$$\text{Density for steel} = 7.83 \times 10^6 \text{ g/m}^3$$

then:

$$J \text{ (g}\cdot\text{m}^2) = D^4 \times l \times 7.7 \times 10^5$$

The reflected inertia of the load is:

$$J \text{ reflected (g}\cdot\text{m}^2) = M \text{ (load)} L^2 \times .025$$

Total Torque Load from lead screw (T) in mN•m

$$T = (J \text{ rotor} + J \text{ screw} + J \text{ reflected}) \alpha + T \text{ friction}$$

8. Motor watts output

$$\text{Watts out} = \text{Torque output} \times \text{speed in radians/sec}$$

$$1 \text{ watt} = 1 \text{ Nm/sec}$$

For a given output Torque (mN•m) and converting v (steps/sec) to radians/sec

$$\text{Watts out} = \text{Torque (mN}\cdot\text{m)} \times v \frac{(\text{motor step angle})}{57.3} \times 10^{-3}$$

If the speed is in RPM then:

$$\text{Watts out} = 1.05 \times 10^{-4} \times \text{torque (mN}\cdot\text{m)} \times \text{RPM}$$

9. Steps/sec to RPM

$$\text{RPM} = \frac{v \text{ (steps/sec)} \times 60}{\text{motor steps/rev}}$$

For information or to place an order in North America: 1 (203) 271-6444 Europe: (44) 1276-691622 Asia: (65) 7474-888

The specifications in this publication are believed to be accurate and reliable. However, it is the responsibility of the product user to determine the suitability of Thomson products for a specific application. While defective products will be replaced without charge if promptly returned, no liability is assumed beyond such replacement.